



TEXTBOOK SERIES
FROM EUROPA-LEHRMITTEL
for the metalworking trades

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Mechanical and Metal Trades Handbook

4th English edition

VERLAG EUROPA-LEHRMITTEL · Nourney, Vollmer GmbH & Co. KG
Düsselberger Straße 23 · 42781 Haan-Gruiten · Germany

Europa-No.: 1910X

Original title:

Tabellenbuch Metall, 47th edition 2017, 3rd batch

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Most recent editions of standards and other regulations govern their use.

They can be ordered from Beuth Verlag GmbH, Burggrafenstr. 6, 10787 Berlin, Germany.

The content of the chapter "Program structure of CNC machines according to PAL" (page 349 to 368) complies with the publications of the PAL Prüfungs- und Lehrmittelentwicklungsstelle (Institute for the development of training and testing material) of the IHK Region Stuttgart (Chamber of Commerce and Industry of the Stuttgart region).

4th edition 2018, corrected reprint 2021

Batch 5 4 3

All batches of this edition may be used concurrently in the classroom since they are unchanged, except for some corrections to typographical errors and slight changes in standards.

ISBN 978-3-8085-1915-8

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© 2018 by Verlag Europa-Lehrmittel, Nourney, Vollmer GmbH & Co. KG, 42781 Haan-Gruiten, Germany
www.europa-lehrmittel.de

Translation and typesetting: A.C.T. Fachübersetzungen GmbH, 41061 Mönchengladbach, Germany

Typesetting and layout of the German edition: Satz+Layout Werkstatt Kluth GmbH, 50374 Erftstadt, Germany

Cover design: Grafische Produktionen Jürgen Neumann, 97222 Rimpf, Germany

Cover photo: Sauter Feinmechanik GmbH, 72555 Metzingen, Germany

Printed by: mediaprint solutions GmbH, 33100 Paderborn, Germany

Preface

The Mechanical and Metal Trades Handbook is well-suited for shop reference, tooling, machine building, maintenance and as a general book of knowledge. It is also useful for educational purposes, especially in practical work or curricula and continuing education programs.

Target Groups

- Industrial and trade mechanics
- Technical product designers
- Apprentices in above trade areas
- Practitioners in trades and industry
- Mechanical engineering students

Notes for the user

The contents of this book include tables and formulas in seven chapters as well as a table of contents, a subject index, a standard index and an international material comparison chart.

For a better overview, each of the seven chapters is preceded by an additional one-page **table of contents**.

The **tables** contain the most important guidelines, designs, types, dimensions and standard values for their respective subject areas.

Units are not specified in the legends for the **formulas** if several units are possible. However, the calculation examples for each formula use those units normally applied in practice.

Changes in the 4th edition

The standards in this edition are **current as of January 2017**. Due to new standards and technical developments, the following contents have been updated, expanded or newly added:

- Quality management and environmental management according to the latest standard. Elimination of general terms from quality management.
- Introduction to "Geometrical product specification (GPS)" for technical communication principles.
- Additional tools and partially updated standard values for machining operations.
- Additions in cost accounting.
- Representation of structuring principles and reference designation in schematic circuit diagrams according to ISO 1219 and DIN EN 81346.

Authors and publishers continue to be grateful to all users of the table handbook for notes and suggestions for improvement addressed to lektorat@europa-lehrmittel.de.

Summer 2018

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Standards and other regulations

Standardization and standards terms

Standardization is the systematic achievement of uniformity of material and non-material objects, such as components, calculation methods, process flows and services for the benefit of the general public.


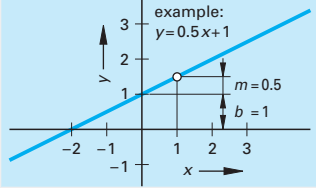
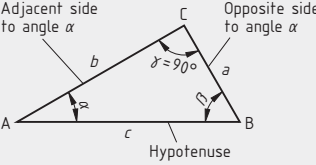


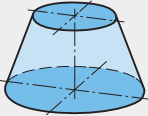
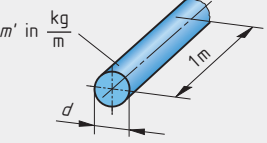
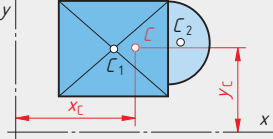
Standards term	Example	Explanation
Standard	DIN 509	A standard is the published result of the standardization work. Example: DIN 509 with shapes and dimensions of undercuts for turned parts and bores.
Part	DIN 30910-2	Standards can comprise several parts associated with each other. The part numbers are appended to the main standard number with hyphens. DIN 30910-2 describes sintered materials for filters for example, whereas Part 3 and 4 deal with sintered materials for bearings and formed parts.
Supplement	DIN 743 Suppl. 1	A supplement contains information for a standard, however no additional specifications. The supplement DIN 743 Suppl. 1, for example, contains application examples of load capacity calculations for shafts and axles described in DIN 743.
Draft	E DIN EN 10027-2 (2013-09)	Draft standards are made available to the public for examination and commenting. The new version of DIN EN 10027-2 (2015-07) with material codes for steels was available to the public as a draft for objections, for example, from September 2013 to February 2014.
Preliminary standard	DIN V 45696-1 (2006-02)	A preliminary standard contains the results of standardization, which have not been released as a standard because of certain provisos. For example, DIN V 45696-1 contains technical measures for the design of machines that transmit whole-body vibrations to humans.
Output date	DIN 76-1 (2004-06)	Date of publication which is made public in the DIN publication guide; this is the date at which time the standard becomes valid. DIN 76-1, which sets undercuts for metric ISO threads has been valid since June 2004 for example.

Types of standards and regulations (selection)

Type	Abbreviation	Explanation	Purpose and contents
International standards (ISO standards)	ISO	International Organization for Standardization, Geneva (O and S are reversed in the abbreviation)	Simplifies the international exchange of goods and services as well as cooperation in scientific, technical and economic areas.
European standards (EN standards)	EN	European Committee for Standardization (Comité Européen de Normalisation), Brussels	Technical harmonization and the associated reduction of trade barriers for the advancement of the European market and the coalescence of Europe.
German standards (DIN standards)	DIN	Deutsches Institut für Normung e.V., Berlin (German Institute for Standardization)	National standardization facilitates rationalization, quality assurance, environmental protection and common understanding in economics, technology, science, management and public relations.
	DIN EN	European standard for which the German version has attained the status of a German standard	
	DIN ISO	German standard for which an international standard has been adopted without change.	
	DIN EN ISO	European standard for which an international standard has been adopted unchanged and the German version has the status of a German standard.	
	DIN VDE	Printed publication of the VDE, which has the status of a German standard.	
VDI guidelines	VDI	Verein Deutscher Ingenieure e.V., Düsseldorf (Association of German Engineers)	These guidelines give an account of the current state of the art in specific subject areas and contain, for example, concrete procedural guidelines for the performing calculations or designing processes in mechanical or electrical engineering.
VDE printed-publications	VDE	Verband der Elektrotechnik (German Association of Electrical Engineering) Elektronik Informationstechnik e.V., Frankfurt am Main (Association for Electronics Information Technology)	Recommendations in the area of quality technology.
DGQ publications	DGQ	Deutsche Gesellschaft für Qualität e.V., Frankfurt (German Society for Quality)	Recommendations in the area of production and work planning.
REFA sheets	REFA	Verband für Arbeitsstudien (Association for Work Design) REFA e.V., Darmstadt (Industrial Organization and Corporate Development)	Recommendations in the area of production and work planning.

1 Technical Mathematics

TM

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	<p>1.2 Formulas Formula symbols, mathematical symbols 13 Formulas, equations, graphs 14 Transformation of formulas 15 Quantities and units 16 Calculation with quantities 17 Percentage and interest calculation 17</p>
	<p>1.3 Angles and triangles Types of angles, Pythagorean Theorem 18 Theorem of intersecting lines 18 Functions of right triangles 19 Functions of oblique triangles 19</p>
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Units of measurement

SI¹⁾ base quantities and base units

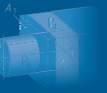
cf. DIN 1301-1 (2010-10), -2 (1978-02), -3 (1979-10)

Base quantity	Length	Mass	Time	Electric current	Thermodynamic temperature	Amount of substance	Luminous intensity
Base units	metre	kilogramme	second	ampere	kelvin	mole	candela
Unit symbol	m	kg	s	A	K	mol	cd

¹⁾ The units for measurement are defined in the International System of Units SI (Système International d'Unités). It is based on the seven basic units (SI units), from which other units are derived.

Base quantities, derived quantities and their units

Quantity	Symbol	Unit Name	Symbol	Relationship	Remarks Examples of application
Length, Area, Volume, Angle					
Length	<i>l</i>	metre	m	1 m = 10 dm = 100 cm = 1000 mm 1 mm = 1000 µm 1 km = 1000 m	1 inch = 25.4 mm In aviation and nautical applications the following applies: 1 international nautical mile = 1852 m
Area	<i>A, S</i>	square metre are hectare	m ² a ha	1 m ² = 10 000 cm ² = 1 000 000 mm ² 1 a = 100 m ² 1 ha = 100 a = 10 000 m ² 100 ha = 1 km ²	Symbol <i>S</i> only for cross-sectional areas Ar and hectare only for land
Volume	<i>V</i>	cubic metre litre	m ³ l, L	1 m ³ = 1000 dm ³ = 1 000 000 cm ³ 1 l = 1 L = 1 dm ³ = 10 dl = 0.001 m ³ 1 ml = 1 cm ³	Mostly for fluids and gases
Plane angle (angle)	$\alpha, \beta, \gamma \dots$	radian degrees minutes seconds	rad ° ' "	1 rad = 1 m/m = 57.2957...° = 180°/π 1° = $\frac{\pi}{180}$ rad = 60' 1' = 1°/60 = 60" 1" = 1°/60 = 1°/3600	1 rad is the angle formed by the intersection of a circle around the centre of 1 m radius with an arc of 1 m length. In technical calculations instead of $\alpha = 33^\circ 17' 27.6''$ it is better to use $\alpha = 33.291^\circ$.
Solid angle	Ω	steradian	sr	1 sr = 1 m ² /m ²	The solid angle of 1 sr encompasses a sphere of r = 1 m on the surface, which corresponds to the area of a spherical segment of A ₀ = 1 m ² .
Mechanics					
Mass	<i>m</i>	kilogramme gram megagram metric ton	kg g Mg t	1 kg = 1000 g 1 g = 1000 mg 1 t = 1000 kg = 1 Mg 0.2 g = 1 ct	In everyday language, the mass of a solid is also referred to as weight. Mass for precious stones in carat (ct).
Linear mass density	<i>m'</i>	kilogramme per metre	kg/m	1 kg/m = 1 g/mm	For calculating the mass of bars, pro-files, pipes.
Area mass density	<i>m''</i>	kilogramme per square metre	kg/m ²	1 kg/m ² = 0.1 g/cm ²	To calculate the mass of sheet metal.
Density	ρ	kilogramme per cubic metre	kg/m ³	1000 kg/m ³ = 1 metric t/m ³ = 1 kg/dm ³ = 1 g/cm ³ = 1 g/ml = 1 mg/mm ³	Density = Mass of a substance per volume unit For homogenous solids, the density is a location-independent quantity.



Units of measurement

TM

Quantities and units (continued)					
Quantity	Symbol	Unit Name	Symbol	Relationship	Remarks Examples of application
Mechanics					
Moment of inertia, 2nd moment of mass	J	kilogramme x square metre	$\text{kg} \cdot \text{m}^2$	The following applies to homogenous full cylinders with a mass m and radius r : $J = \frac{1}{2} \cdot m \cdot r^2$	The moment of inertia indicates the resistance of a rigid homogenous solid against the change in its rotational movement along the axis of rotation.
Force	F	newton	N	$1 \text{ N} = 1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = 1 \frac{\text{J}}{\text{m}}$	The force 1 N effects a change in velocity of 1 m/s in 1 s in a 1 kg mass.
Weight	F_{Wv}, W			$1 \text{ MN} = 10^3 \text{ kN} = 1\,000\,000 \text{ N}$	
Torque Bend. mom. Tors. mom.	M M_b M_T, T	newton x metre	$\text{N} \cdot \text{m}$	$1 \text{ N} \cdot \text{m} = 1 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$	1 N · m is the moment that a force of 1 N effects with a lever arm of 1 m.
Momentum	p	kilogramme x metre per second	$\text{kg} \cdot \text{m/s}$	$1 \text{ kg} \cdot \text{m/s} = 1 \text{ N} \cdot \text{s}$	The momentum is the product of the mass times velocity. It has the direction of the velocity.
Pressure Mechanical stress	p σ, τ	pascal newton per square millimetre	Pa N/mm^2	$1 \text{ Pa} = 1 \text{ N/m}^2 = 0.01 \text{ mbar}$ $1 \text{ bar} = 100\,000 \text{ N/m}^2 = 10 \text{ N/cm}^2 = 10^5 \text{ Pa}$ $1 \text{ mbar} = 1 \text{ hPa}$ $1 \text{ N/mm}^2 = 10 \text{ bar} = 1 \text{ MN/m}^2 = 1 \text{ MPa}$ $1 \text{ daN/cm}^2 = 0.1 \text{ N/mm}^2$	Pressure refers to the force per unit area. For gage pressure, the symbol p_g is used (DIN 1314). 1 bar = 14.5 psi (pounds per square inch)
Second moment of area	I	metre to the fourth power centimetre to the fourth power	m^4 cm^4	$1 \text{ m}^4 = 100\,000\,000 \text{ cm}^4$	Previously: Geometrical moment of inertia
Energy, Work, Quantity of heat	E, W	joule	J	$1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ W} \cdot \text{s} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$	Joule for all forms of energy, kW·h preferred for electrical energy.
Power, Heat flow	P Φ	watt	W	$1 \text{ W} = 1 \text{ J/s} = 1 \text{ N} \cdot \text{m/s} = 1 \text{ V} \cdot \text{A} = 1 \text{ m}^2 \cdot \text{kg/s}^3$	Power describes the work which is achieved within a specific time.
Time					
Time, Time span, Duration	t	seconds minutes hours day year	s min h d a	$1 \text{ min} = 60 \text{ s}$ $1 \text{ h} = 60 \text{ min} = 3600 \text{ s}$ $1 \text{ d} = 24 \text{ h} = 86\,400 \text{ s}$	3 h means a time span (3 hrs.), 3 ^h means a point in time (3 o'clock). If points in time are written in mixed form, e.g. 3 ^h 24 ^m 10 ^s , the symbol min can be shortened to m.
Frequency	f, ν	hertz	Hz	$1 \text{ Hz} = 1/\text{s}$	1 Hz $\hat{=}$ 1 cycle in 1 second.
Rotational speed, Rotational frequency	n	1 per second 1 per minute	1/s 1/min	$1/\text{s} = 60/\text{min} = 60 \text{ min}^{-1}$ $1/\text{min} = 1 \text{ min}^{-1} = \frac{1}{60} \text{ s}^{-1}$	The number of revolutions per unit of time gives the revolution frequency, also called rpm.
Velocity	v	metre per second metre per minute kilometre per hour	m/s m/min km/h	$1 \text{ m/s} = 60 \text{ m/min} = 3.6 \text{ km/h}$ $1 \text{ m/min} = \frac{1 \text{ m}}{60 \text{ s}}$ $1 \text{ km/h} = \frac{1 \text{ m}}{3.6 \text{ s}}$	Speed for nautical journeys in knots (kn): 1 kn = 1.852 km/h mile per hour = 1 mile/h = 1 mph 1 mph = 1.60934 km/h
Angular velocity	ω	1 per second radians per second	1/s rad/s	$\omega = 2 \pi \cdot n$	For a rpm of $n = 2/\text{s}$ the angular velocity $\omega = 4 \pi/\text{s}$.
Acceleration	a, g	metre per second squared	m/s^2	$1 \text{ m/s}^2 = \frac{1 \text{ m/s}}{1 \text{ s}}$	Symbol g only for acceleration due to gravity. $g = 9.81 \text{ m/s}^2 \approx 10 \text{ m/s}^2$

Units of measurement

Quantities and units (continued)

Quantity	Symbol	Unit Name	Symbol	Relationship	Remarks Examples of application
Electricity and magnetism					
Electric current	I	ampere	A		A moving electrical charge is called electricity. The electromotive force is equal to the potential difference between two points in an electric field. The reciprocal of the electrical resistance is called the electrical conductivity.
Electro-motive force	U	volt	V	$1 \text{ V} = 1 \text{ W}/1 \text{ A} = 1 \text{ J}/\text{C}$	
Elect. resistance	R	ohm	Ω	$1 \Omega = 1 \text{ V}/1 \text{ A}$	
Elect. conductance	G	siemens	S	$1 \text{ S} = 1 \text{ A}/1 \text{ V} = 1/\Omega$	
Specific resistance	ρ	ohm x metre	$\Omega \cdot \text{m}$	$10^{-6} \Omega \cdot \text{m} = 1 \Omega \cdot \text{mm}^2/\text{m}$	$\rho = \frac{1}{\kappa} \text{ in } \frac{\Omega \cdot \text{mm}^2}{\text{m}}$
Conductivity	γ, κ	siemens per metre	S/m		$\kappa = \frac{1}{\rho} \text{ in } \frac{\text{m}}{\Omega \cdot \text{mm}^2}$
Frequency	f	hertz	Hz	$1 \text{ Hz} = 1/\text{s}$ $1000 \text{ Hz} = 1 \text{ kHz}$	Frequency of public electric utility: EU 50 Hz, USA/Canada 60 Hz
Elect. Work	W	joule	J	$1 \text{ J} = 1 \text{ W} \cdot \text{s} = 1 \text{ N} \cdot \text{m}$ $1 \text{ kW} \cdot \text{h} = 3.6 \text{ MJ}$ $1 \text{ W} \cdot \text{h} = 3.6 \text{ kJ}$	In atomic and nuclear physics the unit eV (electron volt) is used.
Phase difference	φ	-	-	for alternating current: $\cos \varphi = \frac{P}{U \cdot I}$	The angle between current and voltage in an inductive or capacitive load.
Elect. field strength	E	volts per metre	V/m		$E = \frac{F}{Q}, C = \frac{Q}{U}, Q = I \cdot t$
Elect. charge	Q	coulomb	C	$1 \text{ C} = 1 \text{ A} \cdot 1 \text{ s}; 1 \text{ A} \cdot \text{h} = 3.6 \text{ kC}$	
Elect. capacity	C	farad	F	$1 \text{ F} = 1 \text{ C}/\text{V}$	
Inductance	L	henry	H	$1 \text{ H} = 1 \text{ V} \cdot \text{s}/\text{A}$	
Power Effective power	P	watt	W	$1 \text{ W} = 1 \text{ J}/\text{s} = 1 \text{ N} \cdot \text{m}/\text{s} = 1 \text{ V} \cdot \text{A}$	In electrical power engineering: Apparent power S in $\text{V} \cdot \text{A}$
Thermodynamics and heat transfer					
Quantity	Symbol	Unit Name	Symbol	Relationship	Remarks Examples of application
Thermodynamic temperature	T, Θ	kelvin	K	$0 \text{ K} = -273.15 \text{ }^\circ\text{C}$	Kelvin (K) and degrees Celsius ($^\circ\text{C}$) are used for temperatures and temperature differences. $t = T - T_0; T_0 = 273.15 \text{ K}$ Conversion in $^\circ\text{F}$: page 51
	Celsius temperature	t, ϑ	degrees Celsius	$^\circ\text{C}$	
Quantity of heat		Q	joule	J	
Net calorific value	H_u	joule per kilogramme joule per cubic metre	J/kg J/m ³	$1 \text{ MJ}/\text{kg} = 1000000 \text{ J}/\text{kg}$ $1 \text{ MJ}/\text{m}^3 = 1000000 \text{ J}/\text{m}^3$	Thermal energy released per kg fuel (or for each m ³) minus the heat of vapourisation of the water vapour contained in the exhaust gases.
Non-SI units					
Length	Area	Volume	Mass	Energy, power	
1 inch (in) = 25.4 mm	1 sq.in = 6.452 cm ²	1 cu.in = 16.39 cm ³	1 oz = 28.35 g	1 PSh = 0.735 kWh	
1 foot (ft) = 0.3048 m	1 sq.ft = 9.29 dm ²	1 cu.ft = 28.32 dm ³	1 lb = 453.6 g	1 PS = 0.7355 kW	
1 yard (yd) = 0.9144 m	1 sq.yd = 0.8361 m ²	1 cu.yd = 764.6 dm ³	1 t = 1000 kg	1 kcal = 4186.8 Ws	
1 nautical mile = 1.852 km	1 acre = 4046.873 m ²	1 gallon (US) = 3.785 l	1 short ton = 907.2 kg	1 kcal = 1.166 Wh	
1 land mile = 1.6093 km	Pressure, force	1 gallon (UK) = 4.546 l	1 Karat = 0.2 g	1 kpm/s = 9.807 W	
		1 barrel (US) = 158.9 l	1 Karat = 0.2 g	1 Btu = 1055 Ws	
		1 barrel (UK) = 159.1 l	1 pound/in ² = 27.68 g/cm ³	1 bhp = 745.7 W	
	1 bar = 14.5 pound/in ²				
	1 N/mm ² = 145.038 pound/in ²				

Formula symbols, mathematical symbols

Formula symbols						cf. DIN 1304-1 (1994-03)
Formula symbol	Meaning	Formula symbol	Meaning	Formula symbol	Meaning	
Length, Area, Volume, Angle						
l	Length	r, R	Radius	α, β, γ	Planar angle	
w	Width	d, D	Diameter	Ω	Solid angle	
h	Height	A, S	Area, Cross-sectional area	λ	Wave length	
s	Linear distance	V	Volume			
Mechanics						
m	Mass	F	Force	G	Shear modulus	
m'	Linear mass density	F_{W}, W	Gravitational force, Weight	μ, f	Coefficient of friction	
m''	Area mass density	T	Torque	W	Section modulus	
ρ	Density	M_T, T	Torsional moment	I	2nd moment of mass	
J	Moment of inertia	M_b	Bending moment	W, E	Work, Energy	
p	Pressure	σ	Normal stress	W_p, E_p	Potential energy	
p_{abs}	Absolute pressure	τ	Shear stress	W_k, E_k	Kinetic energy	
p_{amb}	Ambient pressure	ε	Strain	P	Power	
p_g	Gage pressure	E	Modulus of elasticity	η	Efficiency	
Time						
t	Time, Duration	f, ν	Frequency	a	Acceleration	
T	Cycle duration	v, u	Velocity	g	Local gravitational acceleration	
n	Revolution frequency, Speed	ω	Angular velocity	α	Angular acceleration	
				Q, \dot{V}, q_v	Volumetric flow rate	
Electricity						
Q	Electric charge, Quantity of electricity	L	Inductance	X	Reactance	
E	Electromotive force	R	Resistance	Z	Impedance	
C	Capacitance	ρ	Specific resistance	φ	Phase difference	
I	Electric current	γ, κ	Electrical conductivity	N	Number of turns	
Heat						
T, Θ	Thermodynamic temperature	Q	Heat, Quantity of heat	Φ, \dot{Q}	Heat flow	
$\Delta T, \Delta t, \Delta \theta$	Temperature difference	λ	Thermal conductivity	a	Thermal diffusivity	
t, ϑ	Celsius temperature	α	Heat transition coefficient	c	Specific heat capacity	
α, α	Coefficient of linear expansion	k	Heat transmission coefficient	H_{net}	Net calorific value	
Light, Electromagnetic radiation						
E_v	Illuminance	f	Focal length	I_e	Luminous intensity	
		n	Refractive index	Q_e, W	Radiant energy	
Acoustics						
p	Acoustic pressure	L_p	Acoustic pressure level	N	Loudness	
c	Acoustic velocity	I	Sound intensity	L_N	Loudness level	
Mathematical symbols						cf. DIN 1302 (1999-12)
Math. symbol	Spoken	Math. symbol	Spoken	Math. symbol	Spoken	
\approx	approximately the same	\sim	proportional	\log	logarithm (general)	
$\hat{=}$	corresponds to	a^x	a high x, xth of power of a	\lg	common logarithm	
\dots	and so forth	$\sqrt{\quad}$	square root of	\ln	natural logarithm	
∞	infinity	$\sqrt[n]{\quad}$	nth root of	e	Euler number (e = 2.718281...)	
$=$	equal to	$ x $	amount of x	\sin	sine	
\neq	not equal to	\perp	vertical to	\cos	cosine	
$\stackrel{\text{def}}{=}$	is the same by definition	\parallel	is parallel to	\tan	tangent	
$<$	less than	$\uparrow\uparrow$	parallel in same direction	\cot	cotangent	
\leq	less than or equal to	$\uparrow\downarrow$	parallel in opposite direction	$(), [], \{ }$	parentheses, brackets	
$>$	greater than	\sphericalangle	angle		open and closed	
\geq	greater than or equal to	\triangle	triangle	π	pi (circle constant = 3.14159 ...)	
$+$	plus	\cong	congruent to			
$-$	minus	Δx	delta x (difference between two values)	\overline{AB}	line segment AB	
\cdot	times, multiplied by	$\%$	percent, of a hundred	\widehat{AB}	arc AB	
$-, /, :, \div$	over, divided by, per, to	‰	per mil, of a thousand	a', a''	a prime, a double prime	
Σ	sigma (summation)			a_1, a_2	a sub 1, a sub 2	

Formulas, equations, graphs

Formulas

In most cases, the calculation of physical quantities is done with the help of formulas. They consist of:

- Formula symbols, e.g. v_c for cutting velocity, d for diameter, n for speed
- Operators (calculation rules), e.g. \cdot for multiplication, $+$ for addition, $-$ for subtraction and $\frac{\quad}{\quad}$ (fraction line) for division
- Constants, e.g. π (π) = 3.14159 ...
- Numbers, e.g. 10, 15 ...

The formula symbols (page 13) are wildcards for quantities. When solving mathematical problems, the known quantities with their units are filled in the formulas. Before or during the calculation process, the units are converted in a way that

- the calculation becomes feasible or
- the result comprises the required unit.

Most quantities and units are standardised (page 10).

The **result** is always a **numerical value** accompanied by a **unit**, e.g. 4.5 m, 15 s

Example:

What is the cutting velocity v_c in m/min for $d = 200$ mm and $n = 630$ /min?

$$v_c = \pi \cdot d \cdot n = \pi \cdot 200 \text{ mm} \cdot 630 \frac{1}{\text{min}} = \pi \cdot 200 \text{ mm} \cdot \frac{1 \text{ m}}{1000 \text{ mm}} \cdot 630 \frac{1}{\text{min}} = 395.84 \frac{\text{m}}{\text{min}}$$

Formula for cutting velocity

$$v_c = \pi \cdot d \cdot n$$

Numerical value equations

Numerical value equations or numerical equations are formulas in which the typical conversions of units have already been integrated. The following should be noted when using equations:

The numerical values of the individual quantities may only be used in combination with the designated unit.

- The units are not carried along in the calculation.
- The unit of the quantity to be obtained is predetermined.

Example:

What is the torque T of an electrical motor with a driving power of $P = 15$ kW and a speed of $n = 750$ /min?

$$T = \frac{9550 \cdot P}{n} = \frac{9550 \cdot 15}{750} \text{ N} \cdot \text{m} = 191 \text{ N} \cdot \text{m}$$

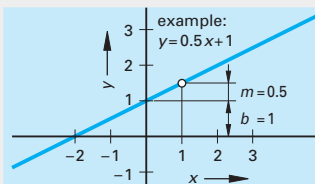
Numerical value equation for torque

$$T = \frac{9550 \cdot P}{n}$$

Designated unit	
Designation	Unit
T	Torque N · m
P	Power kW
n	Speed 1/min

Equations and graphs

In functional equations, y is the function of x , with x as an independent and y as a dependent variable. The number pairs (x, y) of a value table form a graph in the x - y system of coordinates.



1st example:

$$y = 0.5x + 1$$

x	-2	0	2	3
y	0	1	2	2.5

2nd example:

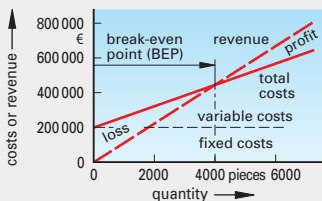
Cost function and revenue function

$$C_t = 60 \text{ €/piece} \cdot Q + 200\,000 \text{ €}$$

$$R = 110 \text{ €/piece} \cdot Q$$

Q	0	4000	6000
C_t	200000	440000	560000
R	0	440000	660000

- C_t total costs \rightarrow dependent variable
- Q quantity \rightarrow independent variable
- C_f fixed costs \rightarrow y coordinate section
- C_v variable costs \rightarrow gradient of the function
- R revenue \rightarrow dependent variable



Assigned function

$$y = f(x)$$

Linear function

$$y = m \cdot x + b$$

Examples:

Cost function

$$C_t = C_v \cdot Q + C_f$$

Revenue function

$$R = R/\text{piece} \cdot Q$$

Transformation of formulas

Transformation of formulas

Formulas and numerical equations are transformed so that the quantity to be obtained stands alone on the left side of the equation. The value of the left side and right side of the formula must not change during the transformation. The following rule applies to all steps of the formula transformation:

Changes applied to the left formula side

= Changes applied to the right formula side

Formula

$$P = \frac{F \cdot s}{t}$$

left side of the formula = right side of the formula

TM

To be able to trace each step of the transformation, it is useful to mark it to the right next to the formula:

$| \cdot t$ → both sides of the formula are multiplied by t .

$| : F$ → both sides of the formula are divided by F .

Transformations of sums

Example: Formula $L = l_1 + l_2$, transformation to find l_2

1	$L = l_1 + l_2$	$ - l_1$	subtract l_1	3	$L - l_1 = l_2$	invert both sides
2	$L - l_1 = l_1 + l_2 - l_1$		perform subtraction	4	$l_2 = L - l_1$	transformed formula

Transformations of products

Example: Formula $A = l \cdot b$, transformation to find l

1	$A = l \cdot b$	$: b$	divide by b	3	$\frac{A}{b} = l$	invert both sides
2	$\frac{A}{b} = \frac{l \cdot b}{b}$		cancel b	4	$l = \frac{A}{b}$	transformed formula

Transformations of fractions

Example: Formula $n = \frac{l}{l_1 + s}$, transformation to find s

1	$n = \frac{l}{l_1 + s}$	$ \cdot (l_1 + s)$	multiply with $(l_1 + s)$	4	$n \cdot l_1 - n \cdot l_1 + n \cdot s = l - n \cdot l_1$	$: n$ subtract divide by n
2	$n \cdot (l_1 + s) = \frac{l \cdot (l_1 + s)}{(l_1 + s)}$		reduce right-side of formula solve the term in brackets	5	$\frac{s \cdot n}{n} = \frac{l - n \cdot l_1}{n}$	cancel n
3	$n \cdot l_1 + n \cdot s = l$	$ - n \cdot l_1$	subtract $- n \cdot l_1$	6	$s = \frac{l - n \cdot l_1}{n}$	transformed formula

Transformations of roots

Example: Formula $c = \sqrt{a^2 + b^2}$, transformation to find a

1	$c = \sqrt{a^2 + b^2}$	$ ()^2$	square the formula	4	$a^2 = c^2 - b^2$	$ \sqrt{\quad}$ square equation
2	$c^2 = a^2 + b^2$	$ - b^2$	subtract b^2	5	$\sqrt{a^2} = \sqrt{c^2 - b^2}$	simplify the expression
3	$c^2 - b^2 = a^2 + b^2 - b^2$		subtract, invert both sides	6	$a = \sqrt{c^2 - b^2}$	transformed formula

Quantities and units

Numerical values and units

Physical quantity

10 mm

Numerical Unit value

Physical quantities, e.g. 125 mm, consist of a

- **Numerical value**, which is determined by measurement or calculation, and a
- **Unit**, e.g. m, kg

Units are standardised in accordance with DIN 1301-1 (page 10).

Very large or very small numerical values can be represented in a simplified way as decimal multiples or factors with the help of prefixes, e.g. 0.004 mm = 4 µm.

Decimal multiples or factors of units

cf. DIN 1301-2 (1978-02)

Symbol	Prefix Name	Power of ten	Mathematical designation	Examples
T	tera	10 ¹²	trillion	12 000 000 000 000 N = 12 · 10 ¹² N = 12 TN (teranewtons)
G	giga	10 ⁹	billion	45 000 000 000 W = 45 · 10 ⁹ W = 45 GW (gigawatts)
M	mega	10 ⁶	million	8 500 000 V = 8.5 · 10 ⁶ V = 8.5 MV (megavolts)
k	kilo	10 ³	thousand	12 600 W = 12.6 · 10 ³ W = 12.6 kW (kilowatts)
h	hecto	10 ²	hundred	500 l = 5 · 10 ² l = 5 hl (hectolitres)
da	deca	10 ¹	ten	32 m = 3.2 · 10 ¹ m = 3.2 dam (decametres)
–	–	10 ⁰	one	1.5 m = 1.5 · 10 ⁰ m
d	deci	10 ⁻¹	tenth	0.5 l = 5 · 10 ⁻¹ l = 5 dl (decilitres)
c	centi	10 ⁻²	hundredth	0.25 m = 25 · 10 ⁻² m = 25 cm (centimetres)
m	milli	10 ⁻³	thousandth	0.375 A = 375 · 10 ⁻³ A = 375 mA (milliamperes)
µ	micro	10 ⁻⁶	millionth	0.000 052 m = 52 · 10 ⁻⁶ m = 52 µm (micrometres)
n	nano	10 ⁻⁹	billionth	0.000 000 075 m = 75 · 10 ⁻⁹ m = 75 nm (nanometres)
p	pico	10 ⁻¹²	trillionth	0.000 000 000 006 F = 6 · 10 ⁻¹² F = 6 pF (picofarads)

Conversion of units

Calculations with physical units are only possible if these units refer to the same base in this calculation. When solving mathematical problems, units often must be converted to basic units, e.g. mm in m, h in s, mm² in m². This is done with the help of conversion factors that represent the value 1 (coherent units).

Conversion factors for units (excerpt)

Quantity	Conversion factors, e. g.	Quantity	Conversion factors, e. g.
Length	$1 = \frac{10 \text{ mm}}{1 \text{ cm}} = \frac{1000 \text{ mm}}{1 \text{ m}} = \frac{1 \text{ m}}{1000 \text{ mm}} = \frac{1 \text{ km}}{1000 \text{ m}}$	Time	$1 = \frac{60 \text{ min}}{1 \text{ h}} = \frac{3600 \text{ s}}{1 \text{ h}} = \frac{60 \text{ s}}{1 \text{ min}} = \frac{1 \text{ min}}{60 \text{ s}}$
Areas	$1 = \frac{100 \text{ mm}^2}{1 \text{ cm}^2} = \frac{100 \text{ cm}^2}{1 \text{ dm}^2} = \frac{1 \text{ cm}^2}{100 \text{ mm}^2} = \frac{1 \text{ dm}^2}{100 \text{ cm}^2}$	Angle	$1 = \frac{60'}{1^\circ} = \frac{60''}{1'} = \frac{3600''}{1^\circ} = \frac{1^\circ}{60 \text{ s}}$
Volume	$1 = \frac{1000 \text{ mm}^3}{1 \text{ cm}^3} = \frac{1000 \text{ cm}^3}{1 \text{ dm}^3} = \frac{1 \text{ cm}^3}{1000 \text{ mm}^3} = \frac{1 \text{ dm}^3}{1000 \text{ cm}^3}$	Inch	1 inch = 25.4 mm; 1 mm = $\frac{1}{25.4}$ inch

1st example:

Convert volume $V = 3416 \text{ mm}^3$ to cm^3 .

Volume V is multiplied by a conversion factor. Its numerator has the unit cm^3 and its denominator the unit mm^3 .

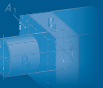
$$V = 3416 \text{ mm}^3 = \frac{1 \text{ cm}^3 \cdot 3416 \text{ mm}^3}{1000 \text{ mm}^3} = \frac{3416 \text{ cm}^3}{1000} = 3.416 \text{ cm}^3$$

2nd example:

The angle size specification $\alpha = 42^\circ 16'$ is to be expressed in degrees ($^\circ$).

The partial angle $16'$ must be converted to degrees ($^\circ$). The value is multiplied by a conversion factor, the numerator of which has the unit degree ($^\circ$) and the denominator the unit minute ($'$).

$$\alpha = 42^\circ + 16' \cdot \frac{1^\circ}{60} = 42^\circ + \frac{16 \cdot 1^\circ}{60} = 42^\circ + 0.267^\circ = 42.267^\circ$$



Calculation with quantities, percentage and interest calculation

Calculation with quantities

Physical quantities are mathematically treated as products.

• Adding and subtracting

Numerical values that have the same unit are added or subtracted and the unit is carried over to the result.

Example:

$$L = l_1 + l_2 - l_3 \text{ where } l_1 = 124 \text{ mm, } l_2 = 18 \text{ mm, } l_3 = 44 \text{ mm; } L = ?$$

$$L = 124 \text{ mm} + 18 \text{ mm} - 44 \text{ mm} = (124 + 18 - 44) \text{ mm} = \mathbf{98 \text{ mm}}$$

• Multiplying and dividing

The numerical values and the units correspond to the factors of products.

Example:

$$F_1 \cdot l_1 = F_2 \cdot l_2 \text{ where } F_1 = 180 \text{ N, } l_1 = 75 \text{ mm, } l_2 = 105 \text{ mm; } F_2 = ?$$

$$F_2 = \frac{F_1 \cdot l_1}{l_2} = \frac{180 \text{ N} \cdot 75 \text{ mm}}{105 \text{ mm}} = 128.57 \frac{\text{N} \cdot \text{mm}}{\text{mm}} = \mathbf{128.57 \text{ N}}$$

• Multiplying and dividing powers

Powers that have the same base are multiplied or divided by adding or subtracting their exponents.

Example:

$$W = \frac{A \cdot a^e}{e} \text{ with } A = 15 \text{ cm}^2, a = 7.5 \text{ cm, } e = 2.4 \text{ cm; } W = ?$$

$$W = \frac{15 \text{ cm}^2 \cdot (7.5 \text{ cm})^2}{2.4 \text{ cm}} = \frac{15 \cdot 56.25 \text{ cm}^{2+2}}{2.4 \text{ cm}} = 351.56 \text{ cm}^{4-1} = \mathbf{351.56 \text{ cm}^3}$$

Rules for raising to powers

a base
 $m, n \dots$ exponents

Multiplying powers

$$a^2 \cdot a^3 = a^{2+3}$$

$$a^m \cdot a^n = a^{m+n}$$

Dividing powers

$$\frac{a^2}{a^3} = a^{2-3}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

Special cases

$$a^{-2} = \frac{1}{a^2}$$

$$a^m = \frac{1}{a^{-m}}$$

$$a^1 = a$$

$$a^0 = 1$$

Percentage calculation

The **percentage rate** indicates the part of the base value in hundredths. The **base value** is the value from which the percentage is to be calculated. The **percent value** is the amount representing the percentage of the base value.

P_r percentage rate, in percent P_v percent value B_v base value

Example:

Weight of raw part: 250 kg (base value); material loss of 2% (percentage rate); material loss in kg = ? (percent value)

$$P_v = \frac{B_v \cdot P_r}{100\%} = \frac{250 \text{ kg} \cdot 2\%}{100\%} = \mathbf{5 \text{ kg}}$$

Percent value

$$P_v = \frac{B_v \cdot P_r}{100\%}$$

Interest calculation

P principle I interest t period in days,
 A amount accumulated r interest rate per year interest period

1st example:

$$P = 2800.00 \text{ €; } r = 6 \frac{\%}{a}; t = \frac{1}{2} a; I = ?$$

$$I = \frac{2800.00 \text{ €} \cdot 6 \frac{\%}{a} \cdot 0.5 a}{100\%} = \mathbf{84.00 \text{ €}}$$

2nd example:

$$P = 4800.00 \text{ €; } r = 5.1 \frac{\%}{a}; t = 50 \text{ d; } I = ?$$

$$I = \frac{4800.00 \text{ €} \cdot 5.1 \frac{\%}{a} \cdot 50 \text{ d}}{100\% \cdot 360 \frac{\text{d}}{a}} = \mathbf{34.00 \text{ €}}$$

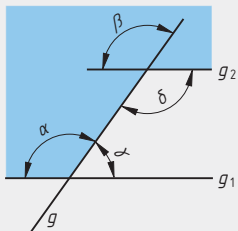
Interest

$$I = \frac{P \cdot r \cdot t}{100\% \cdot 360}$$

1 interest year (1 a) = 360 days (360 d)
360 d = 12 months
1 interest month = 30 days

Types of angles, theorem of intersecting lines, angles in a triangle, pythagorean theorem

Types of angles



- g straight line
 g_1, g_2 parallel straight lines
 α, β corresponding angles
 β, δ opposite angles
 α, δ alternate angles
 α, γ adjacent angles

If two parallels are intersected by a straight line, there are geometrical interrelationships between the resulting angles.

Corresponding angles

$$\alpha = \beta$$

Opposite angles

$$\beta = \delta$$

Alternate angles

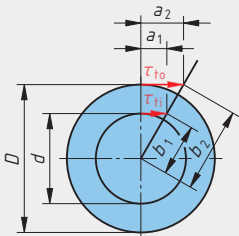
$$\alpha = \delta$$

Adjacent angles

$$\alpha + \gamma = 180^\circ$$

Theorem of intersecting lines

τ_{to} outer torsional stress
 τ_{ti} inner torsional stress



If two intersecting lines are intercepted by a pair of parallels, the resulting segments form equal ratios.

Theorem of intersecting lines

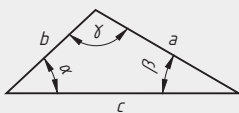
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{d}{D}$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$

$$\frac{b_1}{d} = \frac{b_2}{D}$$

$$\begin{aligned} D &= 40 \text{ mm}, d = 30 \text{ mm}, \\ \tau_{to} &= 135 \text{ N/mm}^2; \tau_{ti} = ? \\ \frac{\tau_{ti}}{\tau_{to}} &= \frac{d}{D} \Rightarrow \tau_{ti} = \frac{\tau_{to} \cdot d}{D} \\ &= \frac{135 \text{ N/mm}^2 \cdot 30 \text{ mm}}{40 \text{ mm}} = 101.25 \text{ N/mm}^2 \end{aligned}$$

Sum of angles in a triangle



- a, b, c sides of the triangle
 α, β, γ angles in the triangle

Example:

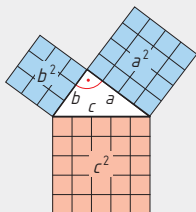
$$\begin{aligned} \alpha &= 21^\circ, \beta = 95^\circ, \gamma = ? \\ \gamma &= 180^\circ - \alpha - \beta = 180^\circ - 21^\circ - 95^\circ = 64^\circ \end{aligned}$$

Sum of angles in a triangle

$$\alpha + \beta + \gamma = 180^\circ$$

In every triangle, the sum of the interior angles equals 180° .

Pythagorean theorem



In a **right triangle** the square on the hypotenuse is equal to the sum of the squares on the sides meeting the right angle.

- a side
 b side
 c hypotenuse

1st example:

$$\begin{aligned} c &= 35 \text{ mm}; a = 21 \text{ mm}; b = ? \\ b &= \sqrt{c^2 - a^2} = \sqrt{(35 \text{ mm})^2 - (21 \text{ mm})^2} = 28 \text{ mm} \end{aligned}$$

2nd example:

$$\begin{aligned} \text{CNC program where } R &= 50 \text{ mm and } l = 25 \text{ mm.} \\ K &= ? \\ c^2 &= a^2 + b^2 \\ R^2 &= l^2 + K^2 \\ K &= \sqrt{R^2 - l^2} = \sqrt{50^2 \text{ mm}^2 - 25^2 \text{ mm}^2} \\ K &= 43.3 \text{ mm} \end{aligned}$$

Length of the hypotenuse

$$c^2 = a^2 + b^2$$

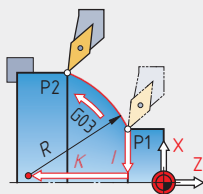
Square on the hypotenuse

$$c = \sqrt{a^2 + b^2}$$

Length of the sides

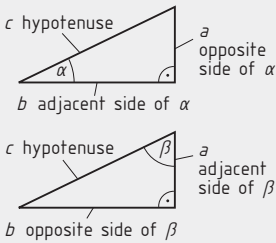
$$a = \sqrt{c^2 - b^2}$$

$$b = \sqrt{c^2 - a^2}$$



Functions of triangles

Functions of right triangles (trigonometric functions)



c hypotenuse (longest side)
 a, b sides:
– b is the adjacent side of α
– a is the opposite side of α
 α, β, γ angles in the triangle, $\gamma = 90^\circ$
 \sin notation of sine
 \cos notation of cosine
 \tan notation of tangent
 $\sin \alpha$ sine of angle α

Trigonometric functions

sine	=	$\frac{\text{opposite side}}{\text{hypotenuse}}$
cosine	=	$\frac{\text{adjacent side}}{\text{hypotenuse}}$
tangent	=	$\frac{\text{opposite side}}{\text{adjacent side}}$
cotangent	=	$\frac{\text{adjacent side}}{\text{opposite side}}$

1st example

$L_1 = 150 \text{ mm}$, $L_2 = 30 \text{ mm}$, $L_3 = 140 \text{ mm}$;
angle $\alpha = ?$

$$\tan \alpha = \frac{L_1 + L_2}{L_3} = \frac{180 \text{ mm}}{140 \text{ mm}} = 1.286$$

Angle $\alpha = 52^\circ$

2nd example

$L_1 = 150 \text{ mm}$, $L_2 = 30 \text{ mm}$, $\alpha = 52^\circ$;
Length of the shock absorber $L = ?$

$$L = \frac{L_1 + L_2}{\sin \alpha} = \frac{180 \text{ mm}}{\sin 52^\circ} = 228.42 \text{ mm}$$

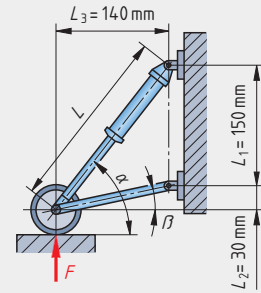
Relations applying to angle α :

$\sin \alpha = \frac{a}{c}$	$\cos \alpha = \frac{b}{c}$	$\tan \alpha = \frac{a}{b}$
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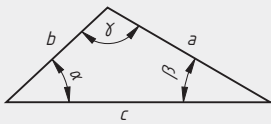
Relations applying to angle β :

$\sin \beta = \frac{b}{c}$	$\cos \beta = \frac{a}{c}$	$\tan \beta = \frac{b}{a}$
----------------------------	----------------------------	----------------------------

The calculation of an angle in degrees ($^\circ$) or as a circular measure (rad) is done with the help of inverse trigonometric functions, e. g. arc sine.



Functions of oblique triangles (law of sines, law of cosines)



According to the law of sines, the ratios of the sides correspond to the sine of their opposite angles in the triangle. If one side and two angles are known, the other values can be calculated with the help of this function.

Side $a \rightarrow$ opposite angle α
Side $b \rightarrow$ opposite angle β
Side $c \rightarrow$ opposite angle γ

Example

$F = 800 \text{ N}$, $\alpha = 40^\circ$, $\beta = 38^\circ$; $F_z = ?$, $F_d = ?$

The forces are calculated with the help of the forces diagram.

$$\frac{F}{\sin \alpha} = \frac{F_z}{\sin \beta} \Rightarrow F_z = \frac{F \cdot \sin \beta}{\sin \alpha}$$

$$F_z = \frac{800 \text{ N} \cdot \sin 38^\circ}{\sin 40^\circ} = 766.24 \text{ N}$$

$$\frac{F}{\sin \alpha} = \frac{F_d}{\sin \varphi} \Rightarrow F_d = \frac{F \cdot \sin \varphi}{\sin \alpha}$$

$$F_d = \frac{800 \text{ N} \cdot \sin 102^\circ}{\sin 40^\circ} = 1217.38 \text{ N}$$

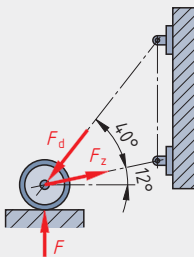
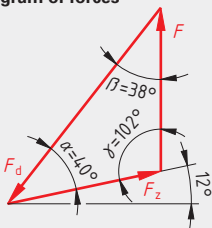


Diagram of forces



Law of sines

$$a : b : c = \sin \alpha : \sin \beta : \sin \gamma$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

There are many transformation options:

$$a = \frac{b \cdot \sin \alpha}{\sin \beta} = \frac{c \cdot \sin \alpha}{\sin \gamma}$$

$$b = \frac{a \cdot \sin \beta}{\sin \alpha} = \frac{c \cdot \sin \beta}{\sin \gamma}$$

$$c = \frac{a \cdot \sin \gamma}{\sin \alpha} = \frac{b \cdot \sin \gamma}{\sin \beta}$$

Law of cosines

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha$$

$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \beta$$

$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos \gamma$$

Transformation, e. g.

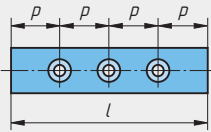
$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$$

The calculation of an angle in degrees ($^\circ$) or as a circular measure (rad) is done with the help of inverse trigonometric functions, e. g. arc cos.

Division of lengths, arc length, composite length

Sub-dividing length

Edge distance = spacing



l total length n number of holes
 p division

Pitch

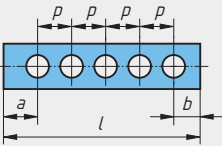
$$p = \frac{l}{n + 1}$$

Example:

$l = 2 \text{ m}; n = 24 \text{ holes}; p = ?$

$$p = \frac{l}{n + 1} = \frac{2000 \text{ mm}}{24 + 1} = 80 \text{ mm}$$

Edge distance \neq spacing



l total length n number of holes
 p spacing a, b edge distances

Pitch

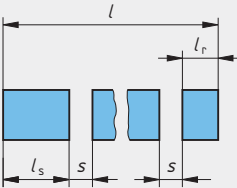
$$p = \frac{l - (a + b)}{n - 1}$$

Example:

$l = 1950 \text{ mm}; a = 100 \text{ mm}; b = 50 \text{ mm};$
 $n = 25 \text{ holes}; p = ?$

$$p = \frac{l - (a + b)}{n - 1} = \frac{1950 \text{ mm} - 150 \text{ mm}}{25 - 1} = 75 \text{ mm}$$

Subdividing into pieces



l bar length s saw cutting width
 z number of pieces l_r remaining length
 l_s piece length

Number of pieces

$$z = \frac{l}{l_s + s}$$

Example:

$l = 6 \text{ m}; l_s = 230 \text{ mm}; s = 1.2 \text{ mm}; z = ?; l_r = ?$

$$z = \frac{l}{l_s + s} = \frac{6000 \text{ mm}}{230 \text{ mm} + 1.2 \text{ mm}} = 25.95 = 25 \text{ pieces}$$

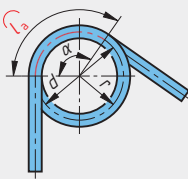
Remaining length

$$l_r = l - z \cdot (l_s + s)$$

$$l_r = l - z \cdot (l_s + s) = 6000 \text{ mm} - 25 \cdot (230 \text{ mm} + 1.2 \text{ mm}) = 220 \text{ mm}$$

Arc length

Example: Torsion spring



l_a arc length α angle at centre
 r radius d diameter

Arc length

$$l_a = \frac{\pi \cdot r \cdot \alpha}{180^\circ}$$

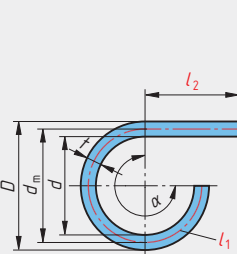
$$l_a = \frac{\pi \cdot d \cdot \alpha}{360^\circ}$$

Example:

$r = 36 \text{ mm}; \alpha = 120^\circ; l_a = ?$

$$l_a = \frac{\pi \cdot r \cdot \alpha}{180^\circ} = \frac{\pi \cdot 36 \text{ mm} \cdot 120^\circ}{180^\circ} = 75.36 \text{ mm}$$

Composite length



D outside diameter d inside diameter
 d_m mean diameter t thickness
 l_1, l_2 section lengths L composite length
 α angle at centre

Composite length

$$L = l_1 + l_2 + \dots$$

Example (composite length, picture left):

$D = 360 \text{ mm}; t = 5 \text{ mm}; \alpha = 270^\circ; l_2 = 70 \text{ mm};$

$d_m = ?; L = ?$

$$d_m = D - t = 360 \text{ mm} - 5 \text{ mm} = 355 \text{ mm}$$

$$L = l_1 + l_2 = \frac{\pi \cdot d_m \cdot \alpha}{360^\circ} + l_2$$

$$= \frac{\pi \cdot 355 \text{ mm} \cdot 270^\circ}{360^\circ} + 70 \text{ mm} = 906.45 \text{ mm}$$